Simultaneous equations: linear and quadratic

ANSWERS

1

Solve

$y = x^2 - 5$

$y = x + 1$

put them equal, then = 0

$x^2 - x - 6 = 0$

factorise

$(x - 3)(x + 2) = 0$

each x, then its y from the line

$x =$ 3 $y =$ 4

$x =$ −2 $y =$ −1

Answer (3, 4) and (−2, −1)

2

Solve

$y = x^2 + 2x$

$y = 4x + 3$

put them equal, then = 0

$x^2 - 2x - 3 = 0$

factorise

$(x - 3)(x + 1) = 0$

each x, then its y from the line

$x =$ 3 $y =$ 15

$x =$ −1 $y =$ −1

Answer (3, 15) and (−1, −1)

3

Solve

$y = x^2 - 3x + 1$

$y = x - 2$

put them equal, then = 0

$x^2 - 4x + 3 = 0$

factorise

$(x - 3)(x - 1) = 0$

each x, then its y from the line

$x =$ 3 $y =$ 1

$x =$ 1 $y =$ −1

Answer (3, 1) and (1, −1)

4

Solve

$y = x^2 + x - 7$

$y = 2x + 5$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x =$ 4 $y =$ 13

$x =$ −3 $y =$ −1

Answer (4, 13) and (−3, −1)

5

Solve

$x^2 + y^2 = 5$

$y = x + 1$

$x^2 + (x + 1)^2 = 5$

$2x^2 + 2x - 4 = 0$

$x^2 + x - 2 = 0$

$(x - 1)(x + 2) = 0$

$x =$ 1 $y =$ 2

$x =$ −2 $y =$ −1

Answer (1, 2) and (−2, −1)

6

Solve

$x^2 + y^2 = 29$

$y = x + 3$

$x^2 + (x + 3)^2 = 29$

$2x^2 + 6x - 20 = 0$, so $x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

Answer (2, 5) and (−5, −2)

7

Nadia solves $y = x^2 - 2x - 4$ and $y = 2x + 1$. She writes (5, −1) and (−1, 11). Check her answers in the line. Explain her mistake and correct it.

$x = 5$ or $x = -1$ is right

but $y = 2(5) + 1 = 11$, not −1

each y goes with its own x

Answer (5, 11) and (−1, −1)

8

Show that the line $y = 4x$ touches the curve $y = x^2 + 4$ at one point only, and find that point.

$x^2 + 4 = 4x$, so $x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$: only $x = 2$

$y = 4 \times 2 = 8$

Answer (2, 8)

Simultaneous equations: linear and quadratic

further practice

ANSWERS

9

Solve $y = x^2 - 6$ and $y = -x$

$x^2 - 6 = -x$, so $x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

Answer (2, −2) and (−3, 3)

10

Solve $y = x^2 + 3x$ and $y = 5x$

$x^2 + 3x = 5x$, so $x^2 - 2x = 0$

$x(x - 2) = 0$, so $x = 0$ or $x = 2$

Answer (2, 10) and (0, 0)

11

Solve $x^2 + y^2 = 34$ and $y = x + 2$

$x^2 + (x + 2)^2 = 34$

$2x^2 + 4x - 30 = 0$, so $x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

Answer (3, 5) and (−5, −3)

12

Solve $y = x^2 - 2x - 1$ and $x + y = 5$

$y = 5 - x$, so $x^2 - 2x - 1 = 5 - x$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

Answer (3, 2) and (−2, 7)

13

On a map of a park, a path follows the line $y = x + 6$ and a stream follows the curve $y = x^2 - 2x + 2$.
Find the coordinates of the points where the path crosses the stream.

$x^2 - 2x + 2 = x + 6$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

Answer (4, 10) and (−1, 5)

14

The line $y = 2x + k$ meets the curve $y = x^2 + 3$ at the point (3, 12). Find k and the other point where they meet.

$12 = 6 + k$, so $k = 6$

$x^2 + 3 = 2x + 6$

$x^2 - 2x - 3 = 0$, $(x - 3)(x + 1) = 0$

Answer $k = 6$, (−1, 4)

15

Solve $x^2 + y^2 = 10$ and $y = 2x + 5$

$x^2 + (2x + 5)^2 = 10$

$5x^2 + 20x + 15 = 0$

$\div 5$: $x^2 + 4x + 3 = 0$

$(x + 1)(x + 3) = 0$

Answer (−1, 3) and (−3, −1)

16

The line $y = k$ touches the curve $y = x^2 - 6x + 11$ at one point only.
Find k and the point.
Explain how you know.

$x^2 - 6x + 11 - k = 0$ has one answer

so it must be $(x - 3)^2 = 0$

$(x - 3)^2 = x^2 - 6x + 9$, so $11 - k = 9$

Answer $k = 2$, (3, 2)