Iteration

ANSWERS

1

4 d.p. in each box

Use $x_{n+1} = \sqrt[3]{x_n + 15}$ with $x_0 = 3$.
Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = \sqrt[3]{x_n + 15}$$

x_0	x_1	x_2	x_3	x_4	x_5
3	2.6207	2.6022	2.6013		

Solution 2.60 (2 d.p.)

2

Use $x_{n+1} = \sqrt[3]{11 - x_n}$ with $x_0 = 2$.
Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = \sqrt[3]{11 - x_n}$$

x_0	x_1	x_2	x_3	x_4	x_5
2	2.0801	2.0739	2.0744		

Solution 2.07 (2 d.p.)

3

Use $x_{n+1} = \sqrt[3]{2x_n + 12}$ with $x_0 = 3$.
Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = \sqrt[3]{2x_n + 12}$$

x_0	x_1	x_2	x_3	x_4	x_5
3	2.6207	2.5834	2.5797		

Solution 2.58 (2 d.p.)

4

Use $x_{n+1} = \sqrt[3]{4x_n + 11}$ with $x_0 = 3$.
Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = \sqrt[3]{4x_n + 11}$$

2.8439, 2.8179

2.8135, 2.8128

Solution 2.81 (2 d.p.)

5

Show that $x^3 + 3x - 9 = 0$ has a solution between $x = 1$ and $x = 2$.

x	$x^3 + 3x - 9$	positive or negative?
1	−5	negative
2	5	positive

$1 + 3 - 9 = -5$, $8 + 6 - 9 = 5$

So the sign changes

6

Show that $x^3 - 6x - 1 = 0$ can be written as $x = \sqrt[3]{6x + 1}$.

Write every step.

$x^3 = 6x + 1$

cube root of both sides

$x =$ $\sqrt[3]{6x + 1}$

7

Use $x_{n+1} = \frac{6 - x_n^3}{7}$ with $x_0 = 1$ to find x_1, x_2 and x_3. Round each to 3 decimal places.

$$x_{n+1} = \frac{6 - x_n^3}{7}$$

x_0	x_1	x_2	x_3
1	0.714	0.805	0.783

$x_3 =$ 0.783

8

Leo uses $x_{n+1} = \sqrt[3]{20 - x_n}$ with $x_0 = 3$.
Each time he types $\sqrt[3]{20 - 3}$, so he gets 2.5713 again and again, and says the solution is 2.57. Explain his mistake.
Then find the solution to 2 decimal places.

he put x_0 in every time: use Ans

2.5713, 2.5927, 2.5917

Solution 2.59 (2 d.p.)

Iteration *further practice*

ANSWERS

9

Use $x_{n+1} = \sqrt[3]{9 - x_n}$ with $x_0 = 2$ to find a solution to 2 decimal places.
Keep going until two values in a row agree.

1.9129, 1.9208

1.9201

Answer *1.92*

10

Show that $x^3 = 4x + 2$ has a solution between $x = 2$ and $x = 3$.

$x^3 - 4x - 2 = 0$

$x = 2: -2, x = 3: 13$

Answer *the sign changes*

11

Show that $x^3 + 6x - 5 = 0$ can be written as $x = \frac{5 - x^3}{6}$. Then use $x_{n+1} = \frac{5 - x_n^3}{6}$ with $x_0 = 1$ to find x_1, x_2 and x_3 to 3 decimal places.

$6x = 5 - x^3$, so $x = \frac{5 - x^3}{6}$

then use Ans each time

Answer *0.667, 0.784, 0.753*

12

Priya thinks of a number. She cubes it, then adds her number. She gets 40.
Write an equation, then use $x_{n+1} = \sqrt[3]{40 - x_n}$ with $x_0 = 3$ to find her number to 2 decimal places.

$x^3 + x = 40$

3.3322, 3.3222, 3.3225

Answer *3.32*

13

The formula $x_{n+1} = \sqrt[3]{k + x_n}$, with $x_0 = 3$, gives $x_1 = 2$. Find k. Then find x_2 to 4 decimal places.

$2^3 = k + 3$, so $k = 5$

$x_2 = \sqrt[3]{5 + 2}$

Answer *$k = 5, x_2 = 1.9129$*

14

Amir uses $x_{n+1} = \sqrt[3]{x_n + 11}$ with $x_0 = 1$.
Chloe uses the same formula with $x_0 = 4$. Do they get the same solution to 2 decimal places?

Amir: 2.2894, 2.3687, 2.3734

Chloe: 2.4662, 2.3791, 2.3740, 2.3737

Answer *yes, both get 2.37*

15

Show that $x^3 + 4x - 2 = 0$ can be written as $x = \frac{2}{x^2 + 4}$. Then use $x_{n+1} = \frac{2}{x_n^2 + 4}$ with $x_0 = 0.5$ to find x_3 to 3 decimal places.

$x(x^2 + 4) = 2$

0.471, 0.474, 0.473

Answer *$x_3 = 0.473$*

16

Ben uses $x_{n+1} = \sqrt[3]{x_n + 17}$ with $x_0 = 3$.
He gets $x_1 = 2.7144$ and $x_2 = 2.7014$.
Both are 2.7 to 1 decimal place, so he stops and gives the solution as 2.7 to 2 decimal places. Explain his mistakes, then give the solution.

2.71 and 2.70 to 2 d.p.: keep going

$x_3 = 2.7008$, so 2.70 (keep the zero)

Answer *2.70*