Algebraic proof

ANSWERS

1

Prove that an odd number, the even number after it and the odd number after that add to a multiple of 6.

write the numbers using n

$2n+1$ + $2n+2$ + $2n+3$

collect like terms

= $6n+6$

factorise to show it

= $6(n+1)$

So it is a multiple of 6

2

Prove that the sum of two consecutive multiples of 6 is a multiple of 6.

write the numbers using n

$6n$ + $6n+6$

collect like terms

= $12n+6$

factorise to show it

= $6(2n+1)$

So it is a multiple of 6

3

Prove that the sum of three consecutive multiples of 4 is a multiple of 12.

write the numbers using n

$4n$ + $4n+4$ + $4n+8$

collect like terms

= $12n+12$

factorise to show it

= $12(n+1)$

So it is a multiple of 12

4

Prove that the sum of four consecutive even numbers is a multiple of 4.

$2n$ + $2n+2$ + $2n+4$ + $2n+6$

= $8n+12$

= $4(2n+3)$

So it is a multiple of 4

5

Prove that $(n+4)^2 - n^2$ is a multiple of 8 for every whole number n.

$n^2 + 8n + 16 - n^2$

$8n + 16$

$8(n+2)$

So it is a multiple of 8

6

Ravi wants to prove that the sum of four consecutive odd numbers is a multiple of 8. He writes $1+3+5+7=16$, $3+5+7+9=24$ and $9+11+13+15=48$, so it is true. Explain why this is not a proof. Then prove it.

examples only check those numbers

$(2n+1)+(2n+3)+(2n+5)+(2n+7)$

$= 8n + 16 = 8(n+2)$

So it is a multiple of 8

7

Prove that $(n+5)^2 - (n+4)^2$ is odd for every whole number n.

$n^2 + 10n + 25 - n^2 - 8n - 16$

$2n + 9$

$2(n+4) + 1$

So it is 1 more than an even number

8

Prove that the difference between the squares of two consecutive even numbers is a multiple of 4.

$(2n+2)^2 - (2n)^2$

$8n + 4$

$4(2n+1)$

So it is a multiple of 4

Algebraic proof *further practice*

ANSWERS

9

Prove that the sum of six consecutive numbers is a multiple of 3.

$n + (n + 1) + \ldots + (n + 5)$

$= 6n + 15$

$= 3(2n + 5)$

So it is a multiple of 3

10

Prove that $3(n + 5) - (n + 3)$ is even for every whole number n.

$= 3n + 15 - n - 3$

$= 2n + 12$

$= 2(n + 6)$

So it is 2 × a whole number, so even

11

Zara picks a whole number. She adds 4, multiplies by 3, takes away the number she picked, then takes away 2.
Prove that her answer is always even.

$3(n + 4) - n - 2$

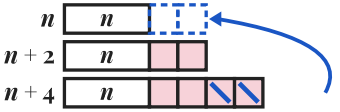
$= 3n + 12 - n - 2 = 2n + 10$

$= 2(n + 5)$

So it is 2 × a whole number, so even

12

The picture shows n, $n + 2$ and $n + 4$ as rows. Use it to explain why their sum is always a multiple of 3.
Then prove it with algebra.



move 2 squares: 3 rows of $n + 2$

sum = $3n + 6 = 3(n + 2)$

So it is a multiple of 3

13

Ella proves that the sum of any two odd numbers is even.
She writes
$(2n + 1) + (2n + 1) = 4n + 2 = 2(2n + 1)$.
What is wrong with her proof?
Write a correct proof.

one n makes both odd numbers equal

$(2n + 1) + (2m + 1) = 2n + 2m + 2$

$= 2(n + m + 1)$

So it is 2 × a whole number, so even

14

Prove that the product of any two odd numbers is odd.

$(2n + 1)(2m + 1)$

$= 4nm + 2n + 2m + 1$

$= 2(2nm + n + m) + 1$

So it is 1 more than an even number

15

Prove that the sum of the squares of two consecutive numbers is always odd.

$n^2 + (n + 1)^2$

$= 2n^2 + 2n + 1$

$= 2(n^2 + n) + 1$

So it is 1 more than an even number

16

Kai says $n^2 + n$ is always even, for every whole number n.
Explain why Kai is right.

$n^2 + n = n(n + 1)$

n and $n + 1$ are consecutive, so one is even

even × a whole number is even

So it is always even